04. MEASURING ANGLES AND TIME

THE SUNDIAL OR SHADOW CLOCK

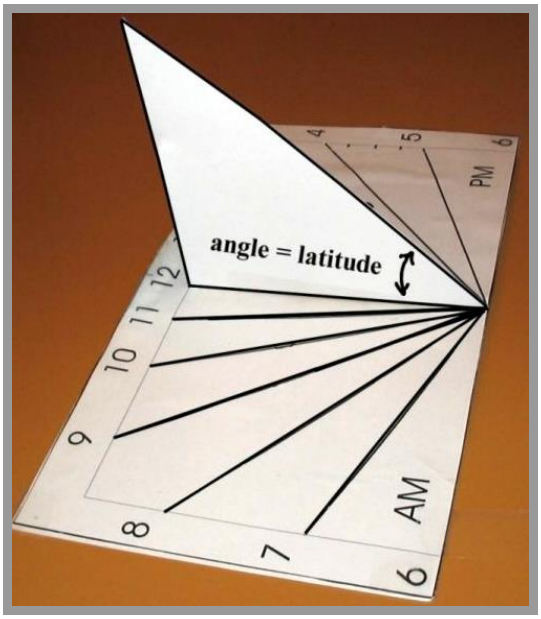
A sundial is a device that tells the time by the cast shadow by a small stick on a graduated plane.

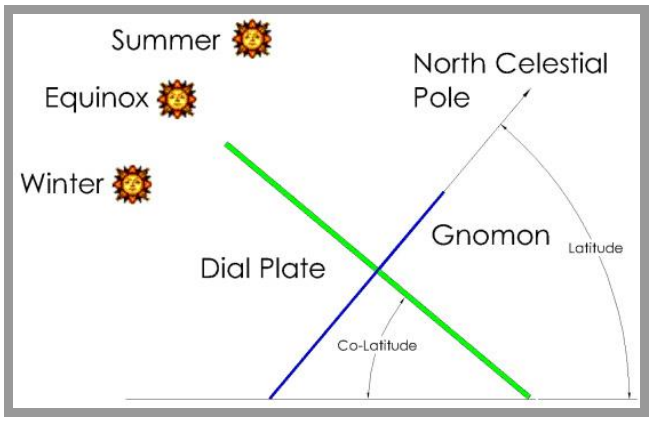
The sundial has:

1. A plane surface with different divisions corresponding to day hours and
2. A gnomon (small stick) that produces the shadow on the surface.

There are different types of sundials: horizontal –place on the floor-, vertical –place on a wall-, equatorial – that has the same slope as the equatorial plane -.

This implies different angles for the gnomon and different angles of separation for each hour.





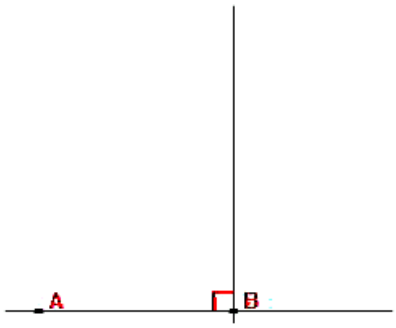
MAKE YOUR OWN HORIZONTAL SUNDIAL

Materials required: A pencil, protractor, compass and a straight edge.

Step 1

Draw on a paper a vertical line; now draw a line perpendicular (AB) to the vertical line. (Fig. #1)

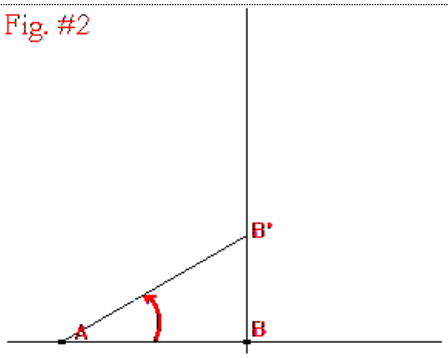
Fig. #1



Step 2

Now draw a line from A through the vertical line so that angle B'AB is equal to the latitude of the place where the sundial is to be used. Remember this angle for making the gnomon. (Fig. #2). 40º for Spain.

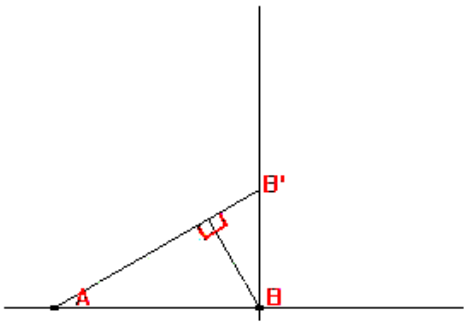
Fig. #2



Step 3

Draw a line perpendicular to line AB' from B. (Fig. #3)

Fig. #3

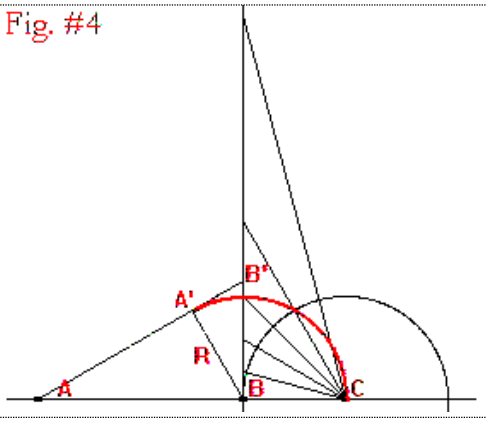


Step 4

Now use the distance A'B to find C (use the compass), draw a semi-circle centered at C and divide it into 15 degree pieces (use the protractor). (Fig. #4)

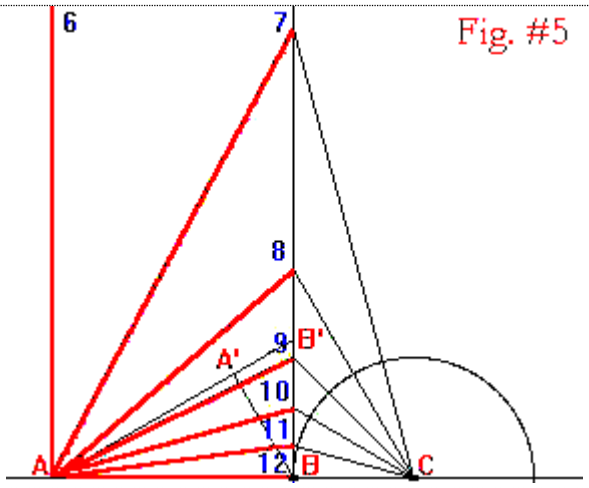
Why 15 degrees? One day is 24 hours. So in 24 hours the sun "moves" through 360 degrees. Therefore 360/24=15 degrees, this is called the hour angle of the sun.

Fig. #4



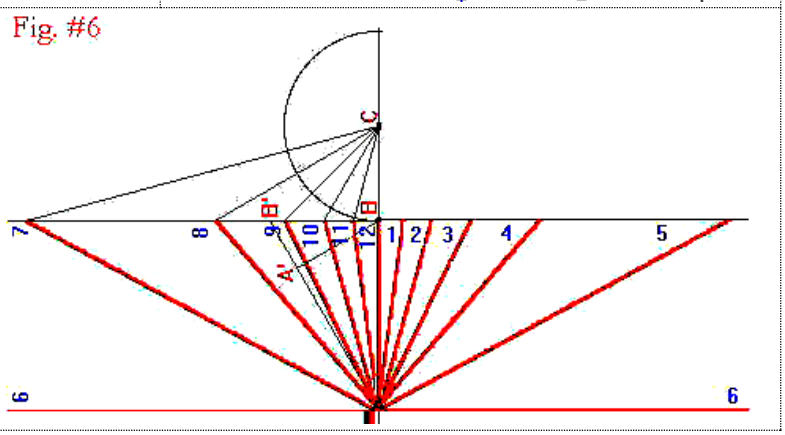
Step 5

Connect the 15 degree points along line BB' to point A. These are the hour lines for your sundial. Draw a line through A and parallel to line BB', this is the 6:00 hour line. (Fig. #5)



Step 6

To find the morning hour line "flip" the afternoon hour lines over the 6 hour line. (Fig. #6)



All that is left is to add the gnomon and set the dial in the sun. The gnomon is a triangular piece that has one angle equal to the latitude of the place where the dial is to be used.

1. MEASURING ANGLES

ANGLE

An angle is the separation between two straight lines that have a common end point (the vertex)

SEXAGESIMAL SYSTEM

A sexagesimal degree is the amplitude of the angle obtained by dividing the circle into 360 equal parts.

A minute is the amplitude of the angle obtained by dividing a degree into 60 equal parts.

A second is the amplitude of the angle obtained by dividing a minute into 60 equal parts.

So 1º=60′; 1′=60″ and 1º=60x60=3600″

And in the opposite way: 1″=1/60′; 1′=1/60º and 1″=1/3600º

Our heart beats about 3600 times per hour.

SEXAGESIMAL AND DECIMAL SYSTEMS

Convert sexagesimal to decimal

For example, convert 35º21′27″ to decimal form.

We convert all units to degrees as follow: $35+\frac{21}{60}+\frac{27}{3600}=35.3631°$

In this case the decimal part represents tenths, hundredths,…of degrees.

Convert decimal to sexagesimal

For example, convert 82,756º to sexagesimal form.

We convert the decimal part to minutes and seconds as follow:

82.756º = 82 + 0.756

0.756º · 60 = 45.36′ = 45 + 0.36

0.36 · 60 = 21.6″ ≈ 22″ (rounded)

So 82.756º = 82º45′22″

2. OPERATIONS WITH ANGLES

Addition

We start by the seconds and so on. We carry the excess to the next unit.

	32º	24′	48″
+	43º	49′	25″

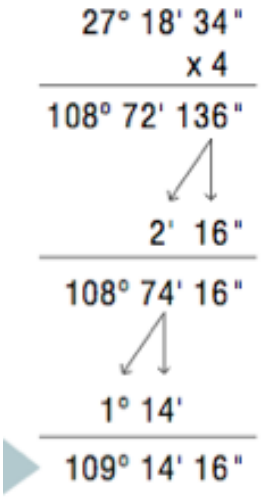
Subtraction

We start by the seconds and so on. We convert a unit to the lower unit if we need it.

	52h	23m	18s
-	43h	49m	25s

Multiplication of an angle by a number

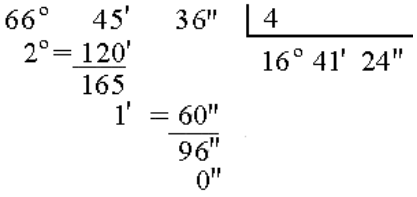
We start multiplying each unit by the number. We carry the excess to the upper unit.



	32º	23′	49″
x			5

Division of an angle by a number

We start dividing the degrees. We convert to a lower unit each remainder.



37º 48' 25" | 5

3. MEASURING TIME

To measure time we use a sexagesimal system too. So we operate in a similar way. We use the following notation of time.

h for hours, m for minutes and s for seconds.

PROBLEMS AND EXERCISES

1. Measuring angles

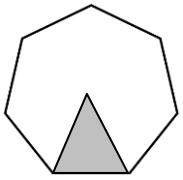
1. Convert to decimal the following sexagesimal angle: 43º 25’ 48”

2. Answer the following questions:
a) Convert to sexagesimal form the following angle: 5.06º
b) Convert to decimal form the following time: 2 h 25 min 23 s.

2. Operations with angles

3. Do the following operations:
a) 57º42’34” + 13º34’51”; b) 130º07’16” – 90º18’25”

4. Find the value of the angles in the shadow of this figure, that is, the angles of the triangle:



5. Determine the supplementary angle of 25° 38' 40"

6. Determine the complementary angle of 38° 36' 43"

7. Answer the following questions:
a) Convert the angle into decimal form: 12° 30' 42".
b) Convert the following time to sexagesimal form: 3,234 hours.

3. Measuring time

8. Calculate:
a) 6 h 53 min 45 s + 7 h 32 min 43 s
b) (11 h 34 min 12 s) × 7

9. Calculate:
a) 5 h 42 min 23 s + 2 h 54 min 35 s + 7 h 46 min 12 s
b) (7 h 12 min 27 s) × 9

10. In a race a car gets 1h27m30s in 50 laps. How long does it take each turn?

11. Transform into sexagesimal form the following time: 5,345 h

12. Do the following operations:
a) 5h53m18s x 8; b) 47h32m51s : 9